



EDGE DETECTION USING FIRST ORDER INFINITE IMPULSE RESPONSE DIGITAL DIFFERENTIATORS

B.T.KRISHNA

Associate Professor

Dept.of Electronics and Communication Engineering
 University College of Engineering Vizianagaram
 Jawaharlal Nehru Technological University Kakinada
 Vizianagaram, Andhra Pradesh, India-535003
 Email:tkbattula@gmail.com

ABSTRACT

The main objective of the paper is to apply first order Infinite Impulse Response (IIR) type digital differentiators for detecting edges of an Image. Initially, design of IIR type digital differentiators and their comparison is presented. Next, the application of first order digital differentiators to various images is discussed. It has been observed that the digital differentiators have shown superior performance compared to the well-known gradient method. The Al-Alaoui digital differentiator has been proved to be superior compared to the other first order differentiators used for the detection of edges of an image.

Index Terms: Digital Integrator, Digital Differentiator, Interpolation, Edge Detection, Filter

I. INTRODUCTION

An edge in an image is a contour across which the brightness of the image changes abruptly [7-10]. Edge detection is an important task in image processing [9-10]. It is a main tool in pattern recognition, image segmentation, and scene analysis. Edge detection refers to the process of identifying and locating sharp discontinuities in an image. There are many ways to perform edge detection. However, the majority of different methods may be grouped into two categories as gradient method and Laplacian method. The gradient method detects the edges by looking for the maximum and minimum in the first derivative of the image. The Laplacian method searches for zero crossings in the second derivative of the image to find edges. The popular edge detection operators are Roberts, Sobel, Prewitt, Frei-Chen, and Laplacian operators etc. Digital differentiators are used to find the time-derivative of the incoming signal [2-6]. A differentiator is defined as, $(\) = \frac{d}{dt}$ where $j = \sqrt{-1}$. These devices are used in almost all fields of engineering like instrumentation, control systems, digital image processing, digital signal processing, bio-medical engineering and other allied fields. It will be a novel idea to apply digital differentiators

for edge detection. The paper is organized as follows. In section 2, Design of digital differentiators of IIR type has been discussed. Application of digital differentiators for the edge detection is presented in section 3. Finally, results and conclusions are presented in section 4.

II. DESIGN OF INFINITE IMPULSE RESPONSE DIGITAL DIFFERENTIATORS

Digital differentiator can be either FIR or an IIR type. IIR type differentiators are preferred to FIR type because of less hardware. An IIR type digital differentiator will be obtained from a digital integrator [2]. An ideal integrator is defined by the following transfer function [3],

$$H(j\omega) = - \quad (1)$$

The commonly used digital integrators are, Backward or rectangular integrator,

$$(\) = - \quad (2)$$

Tustin or trapezoidal integrator,

$$(\) = \frac{(\)}{(\)} \quad (3)$$

In 1992 [2-5], Al-Alaoui has proposed the following procedure for the design of IIR type digital differentiators.

1. Design an integrator that has the same range and accuracy as the desired differentiator.
2. Invert the transfer function of the integrator proposed in step (1) and stabilize it.
3. Compensate the change in magnitude.

2.1 First order Al-Alaoui Digital Differentiator

The First order Integrator is obtained by the Interpolation of Rectangular and trapezoidal Integrators[2-3].So,

$$H(z) = \alpha H_1(z) + (1 - \alpha) H_2(z) \quad (4)$$

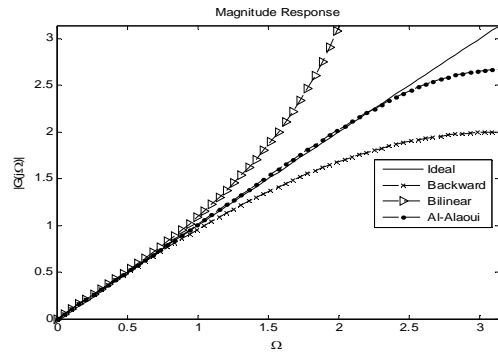
Where α lies between 0 and 1. For $\alpha = 0$ the above equation reduces to,

$$H(z) = H_2(z)$$

(5)

Substituting $H_1(z)$ and $H_2(z)$ in the above equation and simplifying,

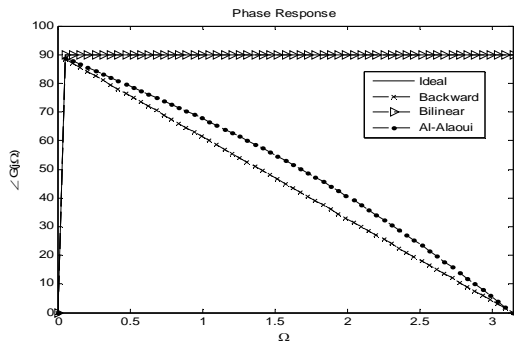
$$H(z) = \frac{1 - \alpha}{1 - \alpha z^{-1}} \quad (6)$$



Reflecting the zero $z = -1$ with its reciprocal $-1/z$, and compensating the magnitude results in a minimum phase digital integrator with the transfer function,

$$H(z) = \frac{1 - \alpha z^{-1}}{1 - \alpha z} \quad (7)$$

Inverting the above transfer function yields the Al-



Alaoui's stabilized IIR differentiator of the first order

$$H(z) = \frac{1 - \alpha}{1 - \alpha z^{-1}} \quad (8)$$

The magnitude and phase responses of the digital differentiators were compared in Fig.1 and 2 respectively. Fig.1.Magnitude Response of the Digital Differentiators

Fig.2.Phase Response of the Digital Differentiators

It can be observed from the graphs that, Al-Alaoui First order differentiator approximates the ideal differentiator till 0.78 of the full band. The lower order of these digital differentiators makes them suitable in real time applications.

III. EDGE DETECTION USING DIGITAL DIFFERENTIATORS

There are many ways to perform edge detection in the Literature[7, 9, 10]. The procedure for the edge detection using IIR type digital differentiators is as follows. The general form of a first order digital differentiator with input $x[n]$ and output $y[n]$ can be written as [2-5],

$$y[n] = \frac{1}{T} (x[n] - x[n-1]) \quad (9)$$

(9)

where, $a = 0$, $k = 1$ for Backward differentiator, $a = 1$, $k = 2$ for Bilinear differentiator, $a = 1/7$, $k = 8/7$ for first order Al - Alaoui differentiator and T is sampling time. The time-domain difference equation can be written as,

$$[G] = -[I] - [I - 1] - [I - 1] \quad (10)$$

Consider an image (I, J) . The Gradient of the image can be written as,

$$\nabla(I, J) = \frac{\partial I}{\partial x} + \frac{\partial J}{\partial y} \quad (11)$$

where \hat{x} , \hat{y} are the unit vectors in x and y directions.

The approximated magnitude of the Gradient is,

$$G = \sqrt{G_x^2 + G_y^2} \quad (12)$$

The following procedure is followed to find the gradient of an image using digital differentiators. Considering $[I] = (I, J)$ and applying it to a digital differentiator individually, and the gradient is calculated using Eqn.(12). The output map is calculated by the procedure mentioned in [8] and is as follows. The magnitude of the gradient $G(x, y, c)$ is evaluated on the function $f(x, y)$ given by the image $\text{mapb}(x, y, c)$ for each color tone c . For each colour, we find the maximum value $G_{\text{Max}}(c)$ on the image map. Let us define the output map as in the following:

$$(I, J) = 255 \frac{G(x, y, c)}{G_{\text{Max}}(c)} \quad (13)$$

where α is a parameter suitable to adjust the image visibility. The figure of merit of edge detectors Root-mean square error is given by,

$$= \frac{1}{N} \sum_{i=1}^N \sum_{j=1}^M (I(i, j) - E(i, j))^2$$

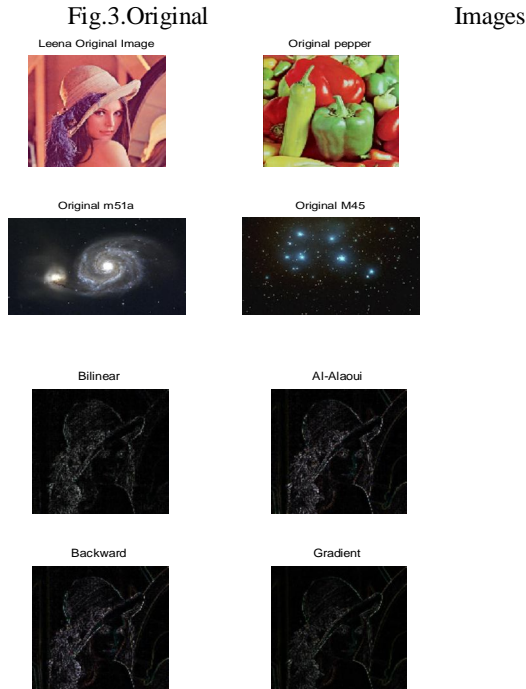
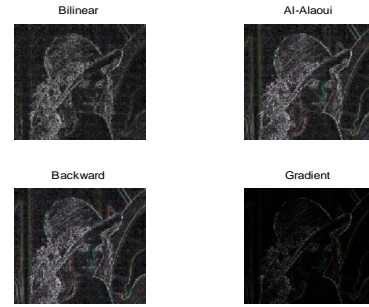
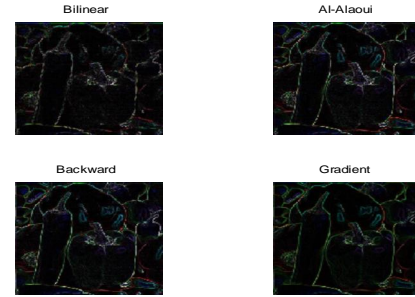
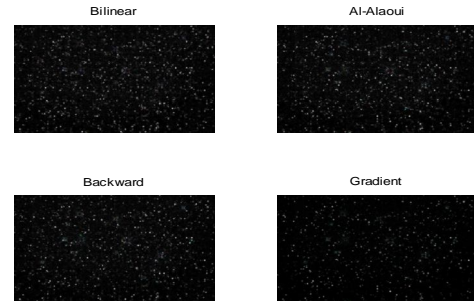
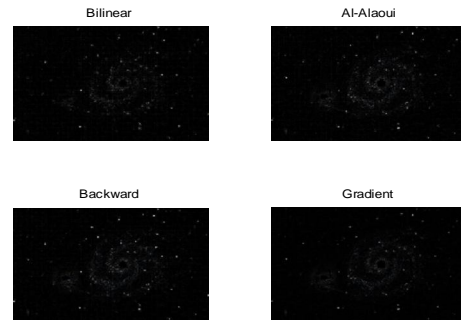
where (I, J) the original is image of size $M \times N$ and (E, F) is the edge detected image. The Root Mean Square values are as shown in Table.1.

Table.1.RMSE of First order digital differentiators

Digital differentiator	RMSE
Backward	0.0024167
Bilinear	0.0025429
Al-Alaoui	0.0024364
Gradient	0.0025219

AND CONCLUSIONS

In this paper, image lenna, pepper, and some of the astronomical images from www.eyetotheuniverse.com has been taken as a test images. Figure 3. Shows the original images. Figure 4. Shows the results of applying digital differentiators using the procedure in Section 3, keeping $\alpha=1$. By varying the brightness constant as 0.5 the results are displayed in Figure 5. Figure 6 shows the obtained results by applying digital differentiators to a pepper image. The results obtained for astronomical images are shown in Figure 8. From the results it can be observed that the performance of the Al-Alaoui digital differentiator is superior compared to other differentiators. Visibility of feeble objects can be enhanced using Al-Alaoui transform to an astronomical image. So, Digital differentiators can be used for the detection of faint objects in astronomical images. Bilinear and backward differentiators also have shown their influence in detecting the edge of an image.

Fig.4.Result after applying digital differentiators at $\alpha = 1$.Fig.5.Result after applying digital differentiators at $\alpha = 0.5$.Fig.6.Result after applying digital differentiators to a pepper image at $\alpha = 1$.Fig.7.Result after applying digital differentiators at $\alpha = 0.75$.Fig.8.Results for $\alpha=1$

REFERENCES

- [1] J.G.Proakis, D.G.Manolakis, *Digital signal processing, principles, algorithms, and applications-3rd edition*, PHI Publications, Newdelhi, 1999.
- [2] M.A.Al-Alaoui, *Novel Digital integrator and Differentiator*-,IEEE *Electronic Letters*, Vol.29, no.4, pp.376-378, Feb.1993.
- [3]M.A.Al-Alaoui, *Novel approach to designing Digital Differentiators*-,IEEE *Electronic Letters*, Vol.28,no.15,pp.1376-1378, Jul.1992.
- [4]M.A.Al-Alaoui, *Novel IIR Digital Differentiator From Simpson Integration Rule*- IEEE *transactions on Circuits Systems.I* ,Fundam Theory Appl., Vol.41,no.2, pp.186-187, Feb.1994.
- [5]J.L.Bihan, *Novel class of Digital integrators and Differentiators*-IEEE *Electronic Letters*, Vol.29, no.11, May.1993.
- [6]Nam QuocNgo, *A New Approach for the Design of Wideband Digital Integrator and Differentiator*-,IEEE *Transactionson Circuits and Systems-II*, Vol.53, No.9, September 2006.
- [7] Gonzalez and Woods, *Digital Image Processing* - Prentice Hall, 2008.
- [8]Amelia Carolina Sparavigna, *Fractional differentiation based image processing*-, arxiv 0910.2381, October 2009.
- [9]J.A.Canny, *Computational Approach to Edge Detector*, IEEE *Transactions on PAMI*, pp679- 698, 1986.
- [10]L.S.Davis, *Edge detection techniques*, *Computer Graphics Image Processing*, Vol.(4),pp.248-270, 1995.